Gradient-Driven Trajectory Deflection in a Density-Based Dynamical Network with Newtonian-Like Scaling

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Abstract

We present a minimal network-based model in which local update rules on a pre-geometric substrate give rise to emergent behaviors quantitatively consistent with weak-field Newtonian gravity. The system consists of interacting nodes on a random geometric graph, characterized by phase and density variables evolving under Kuramoto-type synchronization and density-dependent update rates. A nonlinear reinforcement term with linear decay produces stable, localized high-density clusters at a predictable equilibrium density, serving as mass analogs.

Signal packets propagate via local stochastic rules with no global path optimization. The propagation rule incorporates four factors: update-rate preference, phase alignment, directional bias, and a gradient-coupling term that makes hop probabilities sensitive to the local density gradient. We show that these ingredients produce two distinct geometric effects traceable to specific components of the propagation rule. First, density-dependent update rates generate a scalar cost field that produces systematic travel-time delay for packets passing near clusters (Pearson correlation $r \approx 0.87$ with a post-hoc effective metric), analogous to the Shapiro effect. Second, gradient-sensitive propagation produces spatial deflection of packet trajectories around clusters, with deflection angle scaling as $\theta \propto M/b$ in the weak-field limit, where M is the cluster mass and b is the impact parameter.

The deflection exponent converges toward the exact Newtonian value ($n = 0.92$ at $N = 100$, $n = 0.97$ at $N = 500$), consistent with a well-defined continuum limit. The scaling is mechanistically explained by the radial gradient profile of the density field, which falls off as approximately $1/r^2$. Without gradient coupling, deflection does not depend on impact parameter, identifying gradient sensitivity as the necessary ingredient for spatial curvature in this class of models. Ablation studies confirm that phase dynamics are load-bearing, and null models rule out trivial explanations.

The model does not reproduce general relativity: the effective geometry is scalar rather than tensorial, deflection is repulsive (lensing) rather than attractive (free-fall), and no Lorentz or causal structure is present. These limitations are discussed as explicit directions for future work. The contribution is a minimal, fully transparent computational demonstration that weak-field Newtonian lensing and time delay can emerge from local pre-geometric dynamics, with every step in the causal chain, from microscopic rules to macroscopic scaling law, all analytically and numerically verified.

1. Introduction

A number of approaches in theoretical physics explore the possibility that spacetime is not fundamental but emerges from a deeper pre-geometric structure. These include causal set theory [8], loop quantum gravity [13], causal dynamical triangulations [14], tensor network models [11], and various holographic and information-theoretic frameworks [10]. Motivated by this broad perspective, we consider a minimal computational model in which an underlying substrate of interacting units gives rise to effective geometric behavior through local interaction rules.

In this framework, the substrate is modeled as a network of interacting resonant state-machines characterized by phase and density variables. We investigate the hypothesis that stable, high-density, self-reinforcing patterns within this network act as mass-like objects, and that their presence locally modifies propagation dynamics. Under this interpretation, gravity is not introduced as a fundamental interaction but rather appears as the macroscopic effect of altered propagation rules within the substrate.

This work is independent of holographic or entanglement-based approaches to emergent gravity, but shares their core motivation: that geometric structure may be derived from more fundamental, non-geometric degrees of freedom rather than assumed a priori.

The present work introduces a minimal set of micro-dynamics; resonant oscillators with density-dependent update rates, and tests whether such local rules are sufficient to produce *quantitative* features associated with gravitational behavior. Specifically, we ask:

1. Do local density dynamics produce stable, localized mass-like structures with predictable equilibrium properties?

2. Does the resulting density field generate an effective cost landscape that produces systematic travel-time delay (a Shapiro effect analog)?

3. Under what conditions does signal propagation exhibit spatial deflection with impact-parameter dependence consistent with weak-field Newtonian gravity?

4. Does any observed scaling law survive the continuum limit as the system size increases?

We find that the model produces affirmative answers to all four questions, with the important caveat that spatial deflection requires a specific ingredient; sensitivity of the propagation rule to the local density gradient, that is absent in the simplest formulation. Without gradient coupling, the model produces a scalar conformal factor (position-dependent propagation speed) that generates time delay but not directional deflection. With gradient coupling, deflection scales as $\theta \propto M/b$ (where M is the cluster mass and b is the impact parameter) in the weak-field limit, with the power-law exponent converging toward the exact Newtonian value as the system size increases. This decomposition identifies the minimal ingredients for each class of gravitational phenomenology within a pre-geometric network model.

The model does not attempt to reproduce general relativity: the effective geometry is scalar rather than tensorial, and no Lorentz invariance or causal structure is present. Its contribution is operational, it provides a transparent, fully computable system in which the causal chain from microscopic dynamics to macroscopic scaling law can be traced analytically and verified numerically.

2. Model Definition

We consider a discrete network of interacting units intended to represent a minimal pre-geometric substrate. The system is defined on an undirected graph

$$G = (V, E)$$

where each node $i \in V$ represents a localized degree of freedom and edges $(i, j) \in E$ define interaction locality. The model specifies two layers of dynamics: *substrate evolution*, which governs the formation and persistence of high-density structures, and *signal propagation*, which governs how excitations traverse the substrate. No geometric background is assumed; spatial relationships arise only through the graph topology and the dynamical variables defined on it.

2.1 Node State Variables

Each node carries two dynamical variables: a phase variable $\phi_i(t) \in [0,2\pi)$, representing a local oscillatory state, and a scalar density variable $\rho_i(t) \in [0,1]$, representing local pattern intensity. The density variable serves as a proxy for local pattern concentration and modulates update dynamics.

2.2 Local Update Rate

Each node evolves with a density-dependent update rate given by

$$r_i = r_0(1 - \rho_i)$$

where r_0 is a global baseline rate. This introduces a direct coupling between local density and dynamical timescale: higher-density regions evolve more slowly. The update rate vanishes as $\rho_i \to 1$, producing a natural saturation in dynamical activity at full density. This mechanism is the primary source of travel-time delay in the model and constitutes the analog of gravitational time dilation.

2.3 Phase Dynamics

Phase evolution is governed by a local coupling rule inspired by Kuramoto-type synchronization [1]:

$$\frac{d\phi_i}{dt} = \omega_i + r_i K \sum_{j \in \mathcal{N}(i)} \sin\left(\phi_j - \phi_i\right)$$

where ω_i is the intrinsic frequency of node i, drawn from a bounded uniform distribution $\omega_i \in [-\omega_{\max}, \omega_{\max}]$, K is the coupling strength, and $\mathcal{N}(i)$ denotes the set of neighbors of node i. The factor r_i slows phase evolution in high-density regions, coupling the phase dynamics to the density field.

Phase coherence is not required for propagation alone, but it plays two distinct roles in the model. At the substrate level, synchronization enhances the stability and coherence of high-density clusters. At the propagation level, phase alignment between adjacent nodes modulates hop probabilities and contributes to trajectory structure (Section 2.6). Ablation studies (Section 4) confirm that both contributions are dynamically significant.

2.4 Density Dynamics

The density variable evolves according to

$$\frac{d\rho_i}{dt} = D \sum_{j \in \mathcal{N}(i)} \left(\rho_j - \rho_i\right) + \alpha\, \rho_i^2\, H(\rho_i - \rho_c)\,(1 - \rho_i) - \gamma\, \rho_i$$

where D is the diffusion coefficient, α is the reinforcement strength, ρ_c is the critical density threshold, $H(\cdot)$ is the Heaviside step function, and γ is the linear decay rate. The three terms serve distinct physical roles:

Diffusion ($D \sum_j \left(\rho_j - \rho_i\right)$) smooths density gradients across the network. This is the only term that couples the density at node i to its neighbors, and it acts to spread mass-like structure outward from any concentration. On its own, diffusion would disperse all density variations to a uniform background.

Nonlinear reinforcement ($\alpha\, \rho_i^2\, H(\rho_i - \rho_c)\,(1 - \rho_i)$) stabilizes high-density regions against diffusive loss. The Heaviside factor restricts reinforcement to nodes above the critical threshold ρ_c, preventing low-density regions from self-amplifying. The quadratic dependence on ρ_i ensures that reinforcement strengthens with density, while the saturation factor $(1 - \rho_i)$ prevents density from exceeding unity and removes the numerical singularity that would otherwise arise at $\rho_i = 1$ (where the update rate r_i vanishes).

Linear decay ($-\gamma\, \rho_i$) ensures that density does not persist without active reinforcement. This term is essential for preventing a global cascade in which diffusion-mediated spreading gradually raises all nodes above ρ_c, triggering runaway reinforcement across the entire graph. Without it, the model exhibits global saturation to $\rho_i = 1$ on connected graphs regardless of parameter tuning. With the decay term, only nodes that receive sufficient reinforcement (i.e., those within or near a cluster) maintain high density; isolated nodes relax toward $\rho_i = 0$.

Equilibrium analysis. At steady state, the cluster interior satisfies the balance condition between reinforcement and decay (with diffusion approximately balanced for interior nodes):

$$\alpha\,\rho^*(1-\rho^*) = \gamma$$

This quadratic gives the equilibrium cluster density

$$\rho^* = \frac{1+\sqrt{1-4\gamma/\alpha}}{2}$$

which exists provided $\gamma < \alpha/4$. For the default parameters ($\alpha = 1.0$, $\gamma = 0.10$), this yields $\rho^* \approx 0.89$, in agreement with simulation results at all tested system sizes ($N = 100$ to 500). The background equilibrium is $\rho = 0$: below the critical threshold, only diffusion and decay act, and the density relaxes to zero. The cluster is therefore a localized, self-maintaining structure embedded in a quiescent background; this is the model's analog of a mass concentration.

Remark on the decay term. The density equation in the initial formulation of this model (v4) did not include the decay term or the saturation factor, using instead $d\rho_i/dt = D\sum_j \left(\rho_j - \rho_i\right) + \alpha\,\rho_i^2\,H(\rho_i - \rho_c)$. Systematic testing revealed that this equation exhibits global density cascade: diffusion gradually elevates neighbor densities above ρ_c, triggering reinforcement that propagates across the entire connected graph. The stable localized clusters reported in preliminary runs were artifacts of incomplete numerical integration (ODE solver failure at stiffness boundaries near $\rho = 1$). The decay term resolves this instability while preserving the essential physics of localized self-reinforcement.

2.5 Graph Structure

The interaction graph is constructed as a random geometric graph (RGG): N nodes are distributed uniformly in a bounded two-dimensional domain $[0, L]^2$, and edges are formed between all pairs of nodes within a fixed interaction radius R. This construction enforces approximate locality, nearby nodes interact, distant nodes do not, without introducing an explicit spatial metric. The emergent geometry must arise from the density and phase dynamics, not from the graph embedding.

The interaction radius R is chosen to ensure graph connectivity at each system size. For $N = 100$ with $L = 10$, $R = 1.5$ produces a connected graph with average degree approximately 6–7. For larger systems, R is adjusted to maintain comparable average degree and connectivity.

2.6 Signal Propagation

Signal packets are modeled as discrete excitations that propagate across the network according to local stochastic rules. At each step, a packet at node i transitions to a neighboring node $j \in \mathcal{N}(i)$ with probability

$$P(i \to j) \propto \exp\left(-\lambda_r \frac{1}{r_j}\right) \exp\left(-\lambda_\phi \frac{|\Delta\phi_{ij}|}{\pi}\right) \exp(\lambda_d \cos\theta_{ij}) \exp\left(-\lambda_g\left(\rho_j - \rho_i\right)\right)$$

where $r_j = r_0(1 - \rho_j)$ is the update rate at node j, $|\Delta\phi_{ij}|_\pi$ is the wrapped phase difference between nodes i and j, θ_{ij} is the angular deviation between the direction to node j and the direction to the target, and $\rho_j - \rho_i$ is the density difference between destination and current node. The four factors encode distinct contributions to propagation:

Update-rate preference (λ_r): Packets prefer neighbors with higher update rates (lower cost), directing propagation away from the slowest regions of the substrate.

Phase alignment (λ_ϕ): Packets prefer neighbors that are phase-coherent with the current node, enhancing propagation along locally synchronized channels.

Directional bias (λ_d): Packets prefer neighbors that lie in the general direction of the target. This term ensures that packets make net progress rather than diffusing randomly, and controls the tradeoff between ballistic and diffusive propagation regimes.

Gradient coupling (λ_g): Packets prefer neighbors with *lower* density than the current node; that is, they resist moving uphill in the density field. This term makes the propagation rule directionally sensitive to the local density gradient. It is the critical ingredient for producing impact-parameter-dependent spatial deflection, as opposed to isotropic time delay alone.

The distinction between the update-rate term and the gradient-coupling term is central to the model's phenomenology. The update-rate factor depends on the density *at the destination node* and acts isotropically: all neighbors at a given density level receive the same weight regardless of direction. It produces a scalar conformal factor, a position-dependent propagation speed, that generates travel-time delays but no directional deflection (analogous to the Shapiro effect [7]). The gradient-coupling factor depends on the density *difference* between the current and destination nodes, and therefore acts anisotropically: neighbors on the high-density side of the packet receive different weights than neighbors on the low-density side. This asymmetry generates a net lateral force that deflects trajectories around high-density regions, with deflection that depends on the transverse gradient of the density field. Without the gradient-coupling term ($\lambda_g = 0$), the model produces travel-time delay but spatially flat deflection. With it, deflection acquires dependence on impact parameter consistent with weak-field Newtonian scaling (Section 4).

No global metric, path optimization, or awareness of the density field beyond the immediate neighborhood is imposed during propagation. The packet sees only its neighbors and their local state variables. Trajectories and the effective geometry they define emerge entirely from the accumulation of local stochastic transitions.

2.7 Travel-Time Cost

Each hop incurs a time cost given by

$$\tau_{ij} = \tau_0 \left(1 + \beta_r \frac{1}{r_j} + \beta_\phi \frac{|\Delta\phi_{ij}|}{\pi}\right)$$

where τ_0 is the baseline hop duration, and β_r, β_ϕ control the sensitivity of hop cost to update rate and phase alignment, respectively. Total travel time along a trajectory is the sum of hop costs. This travel time is the primary observable from which the effective metric is inferred post hoc (Section 2.8).

2.8 Effective Metric and Observables

An effective travel-time field is constructed post hoc by assigning each edge a cost $c_{ij} = 1/r_j$ and computing shortest paths in the resulting weighted graph. The correlation between simulated packet travel times and these shortest-path predictions provides a measure of how well the dynamics admit a geometric description.

The following observables are measured:

Substrate structure: Cluster formation (number and location of nodes with $\rho > 0.5$), total cluster density $M = \sum_{i\in\text{cluster}} \rho_i$ (the mass analog), cluster radius (density-weighted RMS distance from centroid), maximum density $\rho_{\max}$, and global phase order parameter.

Propagation metrics: Success rate, total travel time, number of hops, and cumulative turning angle (directional change along discrete paths, serving as a curvature proxy).

Force-law observables: Deflection angle θ as a function of impact parameter b (perpendicular distance from the cluster center to the straight line connecting source and target), closest approach distance, and lateral displacement of trajectory centroid from the straight-line path.

Radial profiles: Effective cost field $1/r_i$ and density gradient magnitude $|\nabla\rho|$ (computed as the mean absolute density difference between adjacent nodes) as functions of distance from the cluster center.

These observables allow quantitative comparison between simulated packet trajectories and the predictions of weak-field Newtonian gravity, in which the deflection angle scales as $\theta \propto M/b$ for a point mass.

3. Simulation Setup

3.1 Graph Construction and Initial Conditions

Simulations were conducted on random geometric graphs with $N = 100$, 200, and 500 nodes distributed uniformly in a bounded two-dimensional domain $[0, L]^2$. The domain size L was scaled with N to maintain approximately constant node density (e.g., $L = 10$ for $N = 100$, $L = 14$ for $N = 200$). Edges were formed between all pairs of nodes within a fixed interaction radius $R = 1.5$, producing connected graphs with average degree approximately 6–7 across all system sizes.

Node phases were initialized uniformly at random in $[0,2\pi)$. Density was initialized at low baseline values ($\rho_i \in [0.05,0.15]$ uniformly) with a localized seed region of n_s nodes (seed fraction n_s/N varied from 5% to 20% depending on the experiment) near the domain center, with seed density $\rho \approx 0.85$ and locally aligned phases (standard deviation $\sigma_\phi = 0.1$ radians around a random center phase). Seeding near the domain center ensures that trajectories can pass on all sides of the cluster, providing a range of impact parameters for force-law extraction.

Intrinsic frequencies ω_i were drawn uniformly from $[-0.5,0.5]$.

3.2 Parameters

The following baseline parameters were used throughout unless otherwise noted:

Parameter	Symbol	Value	Role
Baseline update rate	r_0	1.0	Sets timescale
Reinforcement strength	α	1.0	Cluster self-amplification
Diffusion coefficient	D	0.05	Density spreading
Kuramoto coupling	K	1.2	Phase synchronization
Critical density	ρ_c	0.5	Reinforcement threshold
Decay rate	γ	0.10	Prevents global cascade
Update-rate weight	λ_r	1.0	Hop cost sensitivity
Phase-alignment weight	λ_ϕ	1.0	Phase coherence in hops
Forward bias	λ_d	1.0	Directional preference
Gradient coupling	λ_g	1.0	Density-gradient sensitivity
Hop-cost rate factor	β_r	1.0	Travel-time sensitivity
Hop-cost phase factor	β_ϕ	0.5	Phase contribution to cost
Baseline hop time	τ_0	1.0	Sets time unit

The parameter λ_d (forward bias) was set to 1.0 for the quantitative experiments reported in Sections 4.5–4.8. The gradient-coupling parameter λ_g was varied from 0 (original model, no gradient coupling) to 2.0 in the experiments reported in Section 4.5, with $\lambda_g = 1.0$ used as the default for the continuum-limit and mass-sweep experiments.

The diffusion coefficient and decay rate satisfy the constraint $\gamma < \alpha/4$ required for cluster existence, and were chosen to produce a clear density contrast between the cluster ($\rho^* \approx 0.89$) and the background ($\rho \to 0$) while preventing global cascade. These values were validated across all tested system sizes.

3.3 Numerical Integration

The coupled phase-density ODE system was integrated using explicit Euler stepping with timestep $\Delta t = 0.5$ and total integration time $T = 300$. Phase variables were wrapped to $[0, 2\pi)$ after each step. Density was clamped to $[0,1]$ after each step.

Explicit Euler integration was chosen over adaptive ODE solvers (e.g., LSODA, BDF) after systematic testing revealed that the combined phase-density system becomes numerically stiff as $\rho \to 1$ (where the update rate $r_i \to 0$). Adaptive solvers either fail to converge or produce artifacts (global density saturation) due to overshooting at the stiffness boundary. Explicit Euler with density clamping is stable and produces results consistent with the analytical equilibrium prediction $\rho^* \approx 0.89$. The integration timestep $\Delta t = 0.5$ was verified to produce converged steady-state densities (reducing Δt to 0.1 changed ρ^* by less than 0.5%).

3.4 Experimental Protocol

The following experiments were conducted:

Cluster stability. Verification that seeded high-density regions persist at the predicted equilibrium density under the revised density equation, across all system sizes.

Ablation studies. Three control conditions: (B) phase dependence removed from the propagation rule ($\lambda_\phi = 0$, $\beta_\phi = 0$); (C) phase coupling disabled in the substrate ($K = 0$); and (D) null models using random, constant, and shuffled cost fields.

Force-law extraction (without gradient coupling). At $\lambda_g = 0$, source-target pairs were selected to span a range of impact parameters b relative to the cluster center. For each pair, multiple stochastic packets (typically 15–20) were propagated, and the deflection angle, travel time, closest approach distance, and lateral displacement were recorded per trajectory. Impact-parameter bins were constructed and mean deflection was computed per bin.

Force-law extraction (with gradient coupling). The same protocol at $\lambda_g = 0.5$, 1.0, and 2.0 with $\lambda_d = 1.0$.

Continuum-limit sweep. The force-law extraction protocol was repeated at $N = 100$, 200, and 500 with $\lambda_g = 1.0$, and the power-law exponent n in $\theta = A/b^n$ was extracted at each system size.

Mass sweep. The seed fraction was varied (5%, 10%, 15%, 20%) at $N = 200$ with $\lambda_g = 1.0$ to produce clusters of different mass $M = \sum_{i \in \text{cluster}} \rho_i$. Deflection angle was measured at fixed reference impact parameters and plotted against M.

Radial profile extraction. The effective cost field $1/r_i$ and the mean density gradient magnitude $|\rho_j - \rho_i|$ between adjacent nodes were binned by distance from the cluster centroid and fit to candidate radial profiles ($1/r$, exponential, Gaussian).

Source-target pairs for force-law experiments were selected by stratified sampling across impact-parameter bins to ensure uniform coverage of the b range, excluding nodes within the cluster or too close to the cluster center (minimum distance 2.5 units). Only pairs whose straight-line path passes within the central 80% of the source-target segment near the cluster were retained, ensuring that the impact parameter is well-defined and the trajectory has opportunity to interact with the density field.

All experiments used reproducible seeding via a master random number generator seed, with the full parameter configuration and seed recorded for each run.

4. Results

Simulations were conducted across multiple system sizes ($N = 100, 200, 500$), parameter regimes, and model configurations to evaluate cluster formation, propagation effects, and quantitative scaling behavior. Section 4.1 establishes that stable clusters form and persist under the revised density dynamics. Sections 4.2–4.3 present the qualitative propagation effects and ablation controls, largely consistent with earlier formulations. Section 4.4 identifies a critical negative result: without gradient coupling, deflection does not depend on impact parameter. Sections 4.5–4.8 present the quantitative results obtained with gradient coupling enabled, culminating in the confirmation of weak-field Newtonian scaling $\theta \propto M/b$.

4.1 Cluster Formation and Stability

In runs with seeded initial conditions, localized high-density regions formed and persisted over extended simulation times. With the revised density equation (Section 2.4), clusters stabilize at the predicted equilibrium density $\rho^* \approx 0.89$, in agreement with the analytical prediction from the reinforcement-decay balance condition $\alpha\rho^*(1 - \rho^*) = \gamma$. This agreement was verified at all tested system sizes.

Cluster nodes exhibit strong local phase coherence (order parameter > 0.8) and resist perturbation. The number of nodes in the cluster scales with the seed fraction, while the peak density $\rho_{\max}$ remains stable near ρ^* regardless of cluster size. This behavior confirms that cluster mass $M = \sum_{i \in \text{cluster}} \rho_i$ increases by recruiting more nodes at the equilibrium density, not by increasing the density of individual nodes, consistent with the interpretation of M as an extensive quantity analogous to physical mass.

In addition to seeded clusters, spontaneous formation of secondary high-density regions was observed in several runs, suggesting that the model supports endogenous pattern nucleation under appropriate parameter conditions.

Remark on the decay term. Without the linear decay term ($\gamma = 0$), the density equation exhibits global cascade: diffusion-mediated spreading gradually elevates all connected nodes above ρ_c, triggering runaway reinforcement across the entire graph. This cascade occurs under both explicit Euler integration and adaptive ODE solvers, confirming that it is a property of the dynamical system, not a numerical artifact. With $\gamma = 0.10$, clusters remain localized and the background density relaxes to near zero, producing a well-defined density contrast between cluster and substrate.

4.2 Local Update-Rate Modulation and Effective Cost Field

The density-dependent update rule $r_i = r_0(1 - \rho_i)$ produces pronounced spatial variation in local update rates. In the presence of a stable cluster, the effective propagation cost $1/r_i$ rises from a baseline of approximately 1.0 in the far field to approximately 6–7 at the cluster center.

The radial profile of the cost field was extracted by measuring $1/r_i$ as a function of distance from the cluster centroid. The excess cost above the far-field baseline is best described by a Gaussian profile ($R^2 = 0.85, \sigma \approx 1.4$) or exponential decay ($R^2 = 0.81, \lambda \approx 0.9$). A $1/r$ profile (which would correspond to a Newtonian gravitational potential) provides a substantially poorer fit ($R^2 = 0.60$). This is expected: the cost field is set by the density profile, which is determined by diffusion from a localized source, a process that produces Gaussian or exponential envelopes in two dimensions, not power-law decay.

The density *gradient*, however, follows a different radial scaling. The mean absolute density difference $\left|\rho_j - \rho_i\right|$ between adjacent nodes, binned by distance from the cluster center, falls off as approximately $1/r^{1.85}$ ($R^2 = 0.91$ at $N = 500$). This is close to the $1/r^2$ scaling expected for the gradient of a localized source in two dimensions, and it plays a central role in determining the deflection scaling law (Section 4.6).

4.3 Ablation and Null Model Controls

Three control conditions were evaluated to isolate the contributions of individual model components:

Condition B (no phase in propagation). Removing phase dependence from the transition rule ($\lambda_\phi = 0, \beta_\phi = 0$) while retaining phase coupling in the substrate reduces the success rate from approximately 0.81 to 0.64, increases mean travel time by roughly 18%, and reduces path deflection by approximately 35%. Phase alignment thus contributes significantly to structured propagation.

Condition C (no phase coupling in substrate). Disabling Kuramoto coupling ($K = 0$) reduces the global order parameter to approximately 0.4 and weakens path deflection by roughly 40%. Phase coupling contributes to the formation and maintenance of coherent structures that shape the propagation field.

Null models (random, constant, shuffled cost fields). Under all null conditions, packet propagation resembles a near-random walk, with success rates around 0.22. No organized deflection patterns are observed, and correlation with any $1/r$-based effective metric remains below 0.15. The observed delay and deflection effects therefore depend on the structured density field generated by the system dynamics and do not arise generically from arbitrary cost landscapes.

4.4 Propagation Without Gradient Coupling: Time Delay Without Spatial Deflection

A central finding of this work is that the propagation rule *without* the gradient-coupling term ($\lambda_g = 0$) produces systematic travel-time delay but does not produce impact-parameter-dependent spatial deflection.

Travel-time delay (Shapiro analog). Packet travel times are strongly correlated with post-hoc shortest-path predictions in a $1/r_i$-weighted network (Pearson $r \approx 0.87$). Travel time roughly doubles for trajectories whose straight-line path passes close to the cluster center ($b \approx 0.25$) compared to trajectories that pass far from it ($b \approx 3.75$). This is the model's analog of the Shapiro time delay: signals traversing a region of reduced update rate (the "gravitational well") take longer to arrive.

Flat deflection. Despite the clear travel-time signal, the mean deflection angle θ shows no systematic dependence on impact parameter b. Across all b bins, θ remains approximately constant at ~ 1.0–1.3 radians. Fitting $\theta(b)$ to a Newtonian $1/b$ model yields $R^2 \approx -71$ (catastrophically poor). A power-law fit returns an exponent $n \approx 0.02$, consistent with a constant.

This result was tested at $\lambda_d = 1.8$, 0.8, and 0.5 (progressively weakening the forward bias). In all cases, deflection remained flat. The directional bias is not masking a latent gravitational effect, the effect is genuinely absent.

Diagnosis. The update-rate factor $\exp(-\lambda_r/r_j)$ in the hop probability depends on the density *at* the destination node, not on the density *difference* between the current and destination nodes. It acts isotropically: all neighbors at a given density level receive the same weight regardless of direction. The packet experiences the density field as a position-dependent propagation speed (a scalar conformal factor) rather than as a directional force. This produces the time-delay analog of the g_{00} component of the metric tensor, but not the spatial-curvature analog encoded in the g_{ij} components. Spatial deflection requires sensitivity to the *gradient* of the density field.

4.5 Propagation With Gradient Coupling: Emergence of Spatial Deflection

Introducing the gradient-coupling term $\exp\left(-\lambda_g(\rho_j - \rho_i)\right)$ into the hop probability produces qualitatively different behavior. Packets now preferentially avoid hops that go uphill in the density field, creating a net lateral deflection around high-density regions.

With $\lambda_g = 1.0$ and $\lambda_d = 1.0$ at $N = 100$, the deflection angle θ exhibits clear dependence on impact parameter b, rising from ~ 0.8 rad at large b to ~ 2.1 rad at small b. A $1/b$

(Newtonian) fit yields $R^2 = 0.78$. A power-law fit $\theta = A/b^n$ gives $n \approx 0.92$, close to but distinguishable from the Newtonian value $n = 1$.

Increasing λ_g to 2.0 produces stronger deflection with $R^2 = 0.89$ for the $1/b$ fit, though the success rate decreases modestly ($0.81 \rightarrow 0.68$) because uphill hops are more strongly penalized. The Shapiro delay and ablation/null-model controls remain unchanged, confirming that the gradient-coupling term affects spatial deflection without disrupting the underlying substrate dynamics.

The deflection is *repulsive*: packets bend away from the cluster, not toward it. This is consistent with gravitational lensing (light bending around a mass) rather than gravitational attraction (matter falling toward a mass). The distinction is discussed in Section 5.

4.6 Continuum-Limit Convergence

To test whether the observed scaling reflects continuum-like behavior or discrete lattice artifacts, the deflection experiment was repeated at $N = 100, 200$, and 500 with $\lambda_g = 1.0$, $\lambda_d = 1.0$. At each system size, the domain size was scaled to maintain approximately constant node density, and the deflection angle $\theta(b)$ was extracted and fit to a power law $\theta = A/b^n$.

The results show systematic convergence of the power-law exponent toward the Newtonian value:

N	Exponent n	R^2 (Newtonian $1/b$)
100	0.92	0.84
200	0.94	0.86
500	0.97	0.89

The exponent approaches $n = 1$ monotonically as N increases, the R^2 improves, and the range of impact parameters over which the fit holds extends to larger b values. These trends are consistent with the existence of a well-defined continuum limit in which the deflection law converges to the Newtonian weak-field prediction $\theta \propto 1/b$.

The convergence is mechanistically explained by the radial gradient profile. At $N = 500$, the density gradient falls off as $|\nabla\rho| \propto 1/r^{1.85}$, approaching the $1/r^2$ expected for the gradient of a localized source in two dimensions. In the continuum limit, the deflection angle for a test particle traversing a force field $F(r) \propto 1/r^2$ is given by the transverse impulse integral, which yields $\theta \propto 1/b$. The discrete lattice softens the gradient at small scales (producing $n < 1$), and this softening diminishes as the lattice becomes finer.

4.7 Mass Dependence

In Newtonian gravity, the deflection angle scales linearly with the mass of the deflecting body at fixed impact parameter: $\theta \propto M/b$. The $1/b$ dependence was established in Sections 4.5–4.6. To test the M dependence, the cluster mass was varied by changing the seed fraction from 5% to 20% of nodes at $N = 200$ with all other parameters held fixed. The resulting cluster masses ranged from $M \approx 4.2$ to $M \approx 16.8$.

At fixed reference impact parameters, the deflection angle scales approximately linearly with cluster mass:

Seed fraction	Cluster mass M	θ at small b	θ at mid b
5%	4.2	0.95	0.70
10%	7.1	1.65	1.20
15%	11.4	2.30	1.55
20%	16.8	3.10	2.25

A proportional fit $\theta = a \cdot M$ (forced through the origin) yields $R^2 = 0.87$. A linear fit with intercept $\theta = a \cdot M + c$ gives a small positive intercept ($c \approx 0.2$), consistent with a baseline stochastic deflection that is present even without a cluster. The exponent of the power-law fit $\theta(b)$ also improves with increasing mass ($n = 0.89$ at $M \approx 4.2$ to $n = 0.97$ at $M \approx 16.8$), indicating that larger clusters produce cleaner Newtonian-like scaling.

4.8 Combined Result: Weak-Field Newtonian Scaling

The results of Sections 4.5–4.7 establish both components of the Newtonian weak-field deflection law independently:

$$\theta \propto \frac{1}{b} \quad \text{(confirmed by continuum-limit convergence, } n \to 1\text{)}$$

$$\theta \propto M \quad \text{(confirmed by mass sweep, proportional } R^2 = 0.87\text{)}$$

Together, these imply the combined scaling

$$\theta \propto \frac{M}{b}$$

consistent with the prediction for weak gravitational lensing by a point mass in Newtonian gravity. This scaling emerges from purely local stochastic rules on a pre-geometric network, with no metric, force law, or geometric background imposed. The effective force law arises from the density gradient profile ($|\nabla\rho| \propto 1/r^2$) generated by the substrate dynamics, and the deflection is the integral of the transverse component of this gradient along the packet trajectory.

The model simultaneously produces travel-time delay (Shapiro analog, $r \approx 0.87$) from the scalar cost field, and spatial deflection (lensing analog, $\theta \propto M/b$) from the gradient-coupling term. These correspond, respectively, to the temporal and spatial components of gravitational phenomenology. Their separation into distinct mechanistic contributions, one requiring only a cost field, the other requiring gradient sensitivity, is a structural result of the model.

5. Interpretation and Discussion

This section interprets the quantitative results of Section 4, discusses the mechanistic origin of the emergent scaling law, addresses anticipated objections, situates the model relative to existing approaches to emergent gravity, and identifies specific limitations and directions for future work.

5.1 Two Distinct Geometric Effects From One Substrate

The central finding of this work is that the model produces two distinct geometric effects, each traceable to a specific ingredient in the propagation rule:

Time delay arises from the density-dependent update rate $r_i = r_0(1 - \rho_i)$. High-density regions evolve more slowly, producing a scalar cost field $1/r_i$ that increases travel time for packets passing through or near the cluster. This effect requires no gradient coupling; it is present in the original formulation with $\lambda_g = 0$. In the language of general relativity, it corresponds to the g_{00} component of the metric: a position-dependent lapse function that modulates the rate at which local clocks tick. The strong correlation between simulated travel times and post-hoc shortest paths in the cost-weighted network ($r \approx 0.87$) confirms that this scalar field provides an accurate effective description of the time-delay phenomenology.

Spatial deflection arises from the gradient-coupling term $\exp\left(-\lambda_g(\rho_j - \rho_i)\right)$, which makes the hop probability sensitive to the local density gradient. Without this term, deflection does not depend on impact parameter (Section 4.4). With it, deflection scales as $\theta \propto M/b$ in the weak-field limit, with the power-law exponent converging toward the Newtonian value as the system size increases (Section 4.6). In GR terms, gradient coupling provides the analog of the spatial metric components g_{ij}: it is the ingredient that gives the effective geometry directional structure, producing lateral forces that curve trajectories.

This decomposition is a structural result. It identifies the *minimal* ingredients required for each class of gravitational phenomenology within a pre-geometric network model: a density-dependent propagation speed for time delay, and gradient-sensitive propagation for spatial curvature. Neither ingredient alone produces the full gravitational analog; both are needed.

5.2 Why the Scaling is Newtonian

The $\theta \propto M/b$ scaling is not a free parameter of the model; it is a consequence of the density profile generated by the substrate dynamics. The mechanistic chain is:

1. The density equation (diffusion + localized reinforcement + decay) produces a cluster with a density profile that, in steady state, is approximately Gaussian or exponentially decaying from the cluster center.

2. The *gradient* of this profile, the quantity that the gradient-coupling term responds to, falls off as approximately $1/r^2$ at $N = 500$ (Section 4.2). This is the expected scaling for the gradient of a localized source in two dimensions.

3. The deflection angle is determined by the integral of the transverse gradient along the packet trajectory. For a gradient $\propto 1/r^2$, this integral yields $\theta \propto 1/b$ in the weak-field (large-b) limit.

4. Because the gradient magnitude scales linearly with the total cluster density M (larger clusters produce proportionally steeper gradients at the same distance), the deflection also scales linearly with M.

Each step follows from the previous one, and no step requires fine-tuning. The Newtonian scaling is not imposed, it is inherited from the spatial dimensionality and the functional form of the density dynamics. In a three-dimensional version of the model, diffusion from a localized source would produce a density profile falling off as $1/r$ with a gradient $\propto 1/r^2$, and the deflection integral would be expected to yield $\theta \propto M/b$ as well (the same scaling, for different reasons). The fact that the two-dimensional model already produces the Newtonian result is a consequence of the gradient profile happening to have the right power-law exponent in $d = 2$.

This traceability is a strength of the model. The emergent scaling law is not mysterious, it can be predicted analytically from the density dynamics and verified numerically. A model whose scaling law is understood is more useful than one that produces the right answer for unknown reasons.

5.3 Is the Gradient Term Putting in Gravity by Hand?

A natural objection is that the gradient-coupling term $\exp\left(-\lambda_g\left(\rho_j - \rho_i\right)\right)$ is simply encoding a repulsive force law by construction, and that the Newtonian scaling therefore follows trivially.

This objection conflates two distinct claims. The gradient-coupling term encodes *sensitivity to local structure*: the packet prefers hops that go downhill in density over hops that go uphill. The gradient term introduces local directional sensitivity but does not encode any explicit distance-dependent force law or global scaling behavior. The force law ($\theta \propto 1/b$) is not a property of the gradient term, it is a property of the *density profile* that the substrate dynamics produce. A different density equation (for instance, one that generates an exponential rather than a Gaussian profile) would produce a different gradient, a different deflection integral, and a different scaling exponent, all from the same gradient-coupling term.

To make this concrete: the gradient-coupling term is a nearest-neighbor interaction. The packet at node i sees only the density at its immediate neighbors. It has no information about the cluster center, the cluster mass, or the impact parameter. The $1/b$ and M-dependence emerge from the accumulation of many such local decisions along the trajectory, mediated by the spatially structured density field that the dynamics created. This is emergence in the operational sense, macroscopic scaling from microscopic rules.

The more precise version of the objection is: "Why this functional form for the gradient coupling, and not some other?" The answer is that $\exp(-\lambda_g \cdot \Delta\rho)$ is the simplest monotonic function that penalizes uphill hops, just as $\exp(-\lambda_r / r_j)$ is the simplest monotonic function that penalizes low-update-rate destinations. The choice is motivated by parsimony. It would be valuable to verify that the Newtonian scaling is robust to alternative gradient-coupling forms (e.g., quadratic: $\exp\left(-\lambda_g (\rho_j - \rho_i)^2\right)$). If it is, the result depends on the density profile, not the coupling form. If it is not, the specific functional form matters, and the model's predictive scope is narrower. This test is left for future work.

5.4 Lensing, Not Attraction

The deflection produced by the model is *repulsive*: packets bend away from the cluster center, tracing paths around the high-density region rather than falling into it. This is the analog of gravitational lensing, the deflection of light around a massive body, rather than gravitational attraction — the infall of massive particles toward a mass concentration..

In general relativity, both effects arise from the same metric, but they involve different components and different types of test particles. Light bending is governed primarily by the spatial curvature (the g_{ij} components), while the attraction of massive bodies involves the interplay of temporal and spatial curvature (both g_{00} and g_{ij}). The present model's packets are more analogous to photons than to massive particles: they propagate through the substrate at a speed set by the local cost field and are deflected by the transverse gradient, but they do not experience infall.

Producing gravitational attraction, or the tendency of a free test particle to fall *toward* a mass concentration, would require an additional mechanism: a component of the propagation rule that biases hops toward higher-density regions under certain conditions (for instance, when the packet carries a "mass" attribute that couples to the density field). This is not present in the current model, and its absence should be understood as a scope limitation rather than a failure. The model demonstrates the emergence of one class of gravitational phenomenology (lensing and time delay) while leaving another class (free-fall attraction) for future investigation.

5.5 Universality and Packet Independence

Propagation behavior in this model depends explicitly on both density and phase through the hop probability. Despite this, no significant dependence on packet-specific properties was observed across the quantitative experiments: the $\theta \propto M/b$ scaling holds regardless of the packet's initial phase, starting direction, or intrinsic frequency. All packets traversing the same density field at the same impact parameter experience statistically identical deflection.

This universality is consistent with the equivalence principle in general relativity, which states that all test particles follow the same geodesics in a given gravitational field regardless of their composition. In the model, the universality follows from the fact that the gradient-coupling term depends only on the density field (a property of the substrate), not on any property of the packet. Whether this universality persists under more complex propagation rules; for instance, rules in which packets carry internal state that couples to the substrate, is still an open question.

5.6 Relationship to Existing Approaches

The present model shares conceptual motivations with several existing research programs while differing in its methodology and scope.

Analog gravity [2,3] studies systems in which the equations of motion for perturbations in a condensed-matter medium take the same form as wave equations in curved spacetime. The most developed examples involve sound propagation in flowing fluids, where the effective metric is determined by the flow velocity and speed of sound. The present model differs in that it operates on a discrete graph rather than a continuum medium, and the effective metric is generated dynamically by coupled oscillator-density dynamics rather than prescribed by fluid parameters. The two approaches share the key insight that geometric phenomenology can arise from non-geometric substrates.

Quantum graphity [4] and related approaches model spacetime as a dynamical graph in which edges are created and destroyed according to local rules, with geometry emerging from the resulting graph topology. The present model holds the graph topology fixed and instead generates effective geometry through dynamical variables (phase and density) defined on the nodes. This is a complementary approach: quantum graphity asks how connectivity emerges, while the present model asks how propagation structure emerges on a given connectivity.

Entropic gravity (Verlinde [5]; Jacobson [6]) derives gravitational dynamics from thermodynamic principles, typically involving entropy gradients on holographic screens. While the present model does not invoke thermodynamic reasoning, the density gradient that drives spatial deflection plays a formally analogous role to the entropy gradient in Verlinde's framework: both generate effective forces from the spatial variation of a scalar field. The difference is that the density field here has a concrete dynamical origin (the coupled ODE system), whereas the entropy field in Verlinde's approach is defined through holographic considerations.

The contribution of the present model is operational and computational rather than formal or axiomatic. It does not derive Einstein's equations or claim to provide a fundamental theory of gravity. What it provides is a minimal, explicitly computable system in which the full causal chain; from microscopic update rules to macroscopic scaling law, is transparent and verifiable. This makes it useful as a testbed for questions about what ingredients are necessary for gravitational phenomenology to emerge from pre-geometric dynamics. Related graph-based approaches to fundamental physics include

Trugenberger's quantum graph models [12] and Wolfram's computational universe program [9].

5.7 Limitations

Several important limitations constrain the scope of the present results:

Scalar geometry. The effective geometry consists of a scalar cost field (conformal factor) supplemented by a scalar gradient coupling. In general relativity, the metric is a rank-2 tensor with ten independent components in four dimensions. The present model cannot reproduce direction-dependent effects such as frame dragging, gravitational waves, or the specific factor-of-two relationship between Newtonian and general-relativistic light bending. Extending the hop cost to depend on the angle between the hop direction and the density gradient would introduce rank-2 structure, and this represents the most natural next step toward a richer effective geometry.

Two spatial dimensions. All simulations are conducted in two-dimensional domains. While the $\theta \propto M/b$ scaling is the correct Newtonian weak-field result in any dimension, the density profile and gradient structure would differ in three dimensions. Whether the continuum-limit convergence persists in $d = 3$ has not been tested.

No Lorentz invariance or causal structure. The model has no light cone, no causal ordering of events, and no distinction between spacelike and timelike separations. The propagation rule is stochastic and non-relativistic. Incorporating causal structure; for instance, by restricting hops to forward-directed neighbors in a dynamically defined "time" direction, would be a necessary step for any connection to relativistic physics.

Repulsive deflection only. As discussed in Section 5.4, the model produces lensing but not gravitational attraction. A complete gravitational analog would require a mechanism for infall.

Dependence on gradient-coupling parameter. The gradient-coupling strength λ_g is a free parameter. While the *form* of the scaling law ($\theta \propto M/b$) does not depend on λ_g (it is set by the density profile), the *amplitude* of deflection does. The model does not determine λ_g from first principles. A deeper theory would either derive this coupling from the substrate dynamics or show that it emerges naturally under renormalization.

No quantitative comparison to GR. The model recovers the Newtonian weak-field limit but does not test against general-relativistic predictions (e.g., the factor-of-two enhancement of light bending, perihelion precession, or gravitational redshift beyond the Shapiro delay). Such comparisons would require extending the model to incorporate the tensor structure and causal constraints noted above.

5.8 Future Directions

The results suggest several concrete directions for future work, ordered by increasing ambition:

Robustness tests. Verify that the $\theta \propto M/b$ scaling is independent of the functional form of the gradient coupling (e.g., quadratic vs. linear density difference). Test alternative density dynamics (e.g., cubic reinforcement, non-Heaviside activation functions) and confirm that the scaling depends on the gradient profile rather than the specific micro-rules.

Two-body dynamics. Seed two clusters and characterize their interaction. Do they attract, repel, merge, or ignore each other? If the density field mediates an effective interaction between clusters, this would extend the gravitational analogy from test-particle dynamics to source-source interactions.

Three-dimensional extension. Construct random geometric graphs in $d = 3$ and repeat the force-law extraction. In three dimensions, diffusion from a localized source produces a $1/r$ density profile with a $1/r^2$ gradient, which should yield $\theta \propto M/b$ via the same mechanism as in $d = 2$ but with a different density structure.

Conserved quantities. Search for approximately conserved quantities along packet trajectories. In general relativity, geodesics in symmetric spacetimes admit conserved energy and angular momentum. If a quantity analogous to "cost times velocity" is approximately constant along trajectories in the effective metric, this would strengthen the geodesic interpretation.

Direction-dependent hop costs. Make the hop cost (travel time per step) depend on the angle between the hop direction and the local density gradient. This would promote the effective geometry from a scalar conformal factor to a rank-2 structure, potentially enabling the model to reproduce direction-dependent phenomena such as the distinction between radial and tangential propagation speeds.

Dynamical graph topology. Allow edges to form and dissolve in response to density and phase dynamics, combining the fixed-topology approach of the present model with the dynamical-graph approach of quantum graphity. This would enable the model to explore whether graph topology and effective geometry can co-emerge from the same set of local rules.

6. Conclusion

This work presents a minimal network-based model in which local interaction rules on a pre-geometric substrate give rise to emergent gravitational phenomenology with quantitative scaling consistent with the Newtonian weak-field limit.

The model's substrate consists of phase oscillators with density-dependent update rates on a random geometric graph. A nonlinear reinforcement term with linear decay produces stable, localized high-density clusters at a predictable equilibrium density. These clusters serve as mass analogs: they persist, resist perturbation, and generate a structured propagation field through their influence on local update rates.

Signal packets propagating through this substrate via local stochastic rules exhibit two distinct geometric effects. First, density-dependent update rates produce a scalar cost field that generates systematic travel-time delay (a Shapiro effect analog), confirmed by strong correlation ($r \approx 0.87$) between simulated travel times and a post-hoc effective metric. Second, a gradient-coupling term in the propagation rule, which makes hop probabilities sensitive to local density differences, and produces spatial deflection with impact-parameter dependence consistent with weak-field Newtonian lensing: $\theta \propto M/b$.

The Newtonian scaling is not imposed but emergent. It arises because the density dynamics produce a gradient field that falls off as approximately $1/r^2$, and the deflection is the integral of the transverse component of this gradient along the trajectory. The power-law exponent converges toward the exact Newtonian value as the system size increases ($n = 0.92$ at $N = 100$, $n = 0.97$ at $N = 500$), consistent with the existence of a well-defined continuum limit. Deflection scales linearly with cluster mass at fixed impact parameter (proportional $R^2 = 0.87$), completing both components of the $\theta \propto M/b$ prediction. Ablation studies confirm that phase dynamics contribute to substrate stability and propagation structure, while null models establish that the observed effects require the structured density field generated by the system dynamics.

A critical negative result sharpens the picture: without gradient coupling, the model produces time delay but not impact-parameter-dependent deflection. This identifies gradient sensitivity as the necessary ingredient for spatial curvature in this class of models, and cleanly separates the temporal and spatial components of the emergent geometry.

The model does not reproduce general relativity. The effective geometry is scalar, not tensorial. Deflection is repulsive (lensing), not attractive (free-fall). No Lorentz invariance, causal structure, or light-cone geometry is present. These are not oversights but explicit scope boundaries that define the directions for future work: rank-2 effective metrics via direction-dependent hop costs, three-dimensional extensions, two-body cluster interactions, and the search for conserved quantities along trajectories.

Despite these limitations, the model demonstrates that a unified set of local rules; resonant oscillators with density feedback on a random graph can produce multiple features of gravitational phenomenology within a single framework: stable mass-like structures, propagation delay, and quantitative weak-field lensing. Every step in the causal chain from microscopic update rules to macroscopic scaling law is explicit, analytically understood, and numerically verified. This transparency makes the model useful as a computational testbed for investigating what pre-geometric dynamics are necessary and sufficient for gravitational behavior to emerge.

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